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BOUNDED SOLUTIONS FOR A CERTAIN CLASS OF ELLIPTIC BVPS AND CONTINUOUS DEPENDENCE ON PARAMETERS

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Abstract. Combining the sub- and supersolution methods and the existence results for the radial case we present the existence of non-radial solutions to a certain BVP of elliptic type in exterior domain. We prove the continuous dependence on functional parameters for the radial solutions.

1. Introduction

In this paper we shall investigate the existence of classical nonradial positive solutions for a certain BVP governed by a second order PDE of elliptic type

$$(1)\qquad \begin{cases} -\Delta u(y) = p(y)f(u(y)) + b(y), & \text{for } y \in \Omega, \\ u(y) = 0, & \text{for } ||y|| = 1, \\ \lim\limits_{||y|| \to \infty} u(y) = 0, \end{cases}$$

with $n > 2$, $\Omega := \{y \in R^n : ||y|| > 1\}$, $y = (y_1, ..., y_n) \in \Omega$ and $||y|| = \sqrt{\sum_{i=1}^{n} y_i^2}$.

Similar boundary value problems on unbounded domain have been discussed, among others, in [6], [7], [8] (for systems of equations). These results concern the existence of a positive, exponentially decaying, classical solution (or a pair of positive solutions) for the semilinear elliptic eigenvalue problem

$$(2)\qquad \begin{cases} \mathbf{L}u = \lambda \mathbf{f}(y, u), & \text{for } y \in \Omega, \\ u|_{\partial\Omega} = 0, \\ \lim\limits_{||y|| \to \infty} u(y) = 0, \end{cases}$$

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where $n \geq 2$, Ω is a smooth unbounded domain in R^n, λ is a positive parameter and $\mathbf{L}$ is a uniformly elliptic operator in Ω. The earlier works like [1], [2], [3], [4], [9] were devoted either to special cases of (2) or did not guarantee the existence of nontrivial solutions.

Here we cannot apply the results due to E. S. Noussair and C. A. Swanson because they consider the right-hand side of (2) being a negative function for sufficiently large u and all $y \in \Omega$. In our case, the nonlinearity is positive on $\Omega \times I$, where I is an interval, and we do not impose any condition concerning its behaviour with respect to u at infinity.

2. The existence result for nonradial case

We shall employ together the method of sub- and supersolutions presented in [5] and the existence results for radial solutions to the related problem to obtain a supersolution of (1). Let us recall that a solution u is radially symmetric (or briefly radial), if $u(y)$ depends on $r = ||y||$ only.
By a subsolution of

$$\begin{cases} -\triangle u = F(y, u, \nabla u), & \text{for } y \in \Omega, \\ u(y) = 0, & \text{for } y \in \partial\Omega, \end{cases} \tag{3}$$

we understand $\underline{u} \in C(\overline{\Omega}) \cap C^2(\Omega)$ such that

$$\begin{cases} -\triangle \underline{u} \leq F(y, \underline{u}, \nabla \underline{u}), & \text{for } y \in \Omega, \\ \underline{u}(y) \leq 0, & \text{for } y \in \partial\Omega. \end{cases}$$

Similarly, a supersolution is characterized by the reverse signs in the above definition.

Now we recall the following theorem from [5]:

Theorem 1. *Let Ω be an exterior domain (namely $\Omega = R^n \setminus A$, where $A \neq \emptyset$ is a certain compact subset of R^n). Assume that $F : \Omega \times R \times R^n \to R$ satisfies the following conditions:*

(D1) *for each bounded domain $M \subset \Omega$ there exists a continuous function $\zeta_M : R \to R$ such that for all $y \in M$, $u \in R$ and $z \in R^n$*

$$|F(y, u, z)| \leq \zeta_M(u)(1 + |z|^2),$$

(D2) *F is Hölder continuous ($C^{0,r}$) with respect to (y, u, z) and C^1 with respect to u, z,*

(D3) *there exist sub- and supersolution for (3) denoted by $\underline{u}$ and $\overline{u}$ respectively, such that $\underline{u}(y) \leq \overline{u}(y)$ for any $y \in \Omega$.*

Then there exists at least one solution u such that $\underline{u}(y) \leq u(y) \leq \overline{u}(y)$ for any $y \in \Omega$.

Similar approach has been used in [12] which is devoted to the case when the right-hand side of (1) is a general sublinear function. Here we present analogous results for quite different class of nonlinearities with f being an increasing and piecewise smooth function. So there is no information concerning its behaviour outside a certain interval. We also do not distinguish sub- and superlinear case. On account of this, it is not possible to apply the results obtained in [12] to our problem.

In the sequel, the following assumptions are used:

(bp) $b, p : \Omega \to [0, +\infty)$ are Hölder continuous ($C^{0,r}$) on Ω and $\Omega \ni y \longmapsto p(y)f(0) + b(y)$ is not identically equal to zero;

(f1) there exist $u_1 < 0 < u_2$ such that $f : I \to [0, +\infty)$, where $I = (u_1, u_2)$, is continuously differentiable and nondecreasing;

(f2) there exists $0 < d < u_2$ such that
$$\frac{1}{n-2}\left[f(d)\int_1^{+\infty} l^{n-1}\overline{p}(l)dl + \int_1^{+\infty} l^{n-1}\overline{b}(l)dl\right] \le 4d,$$
where $\overline{p}(l) = \sup\limits_{||y||=l} p(y)$ and $\overline{b}(l) = \sup\limits_{||y||=l} b(y)$ for all $l \ge 1$.

Throughout the paper we assume that the conditions above hold. It is worth noting that Theorem 1 cannot be applied directly, because of the fact that the nonlinearity is not sufficiently smooth and there is no information about the existence of positive supersolutions under the above assumptions. To obtain a supersolution of (1) we employ the scheme presented in [12] and the results for the radial case described in the article [10] devoted to the following problem:

$$\begin{cases} -\Delta u(y) = \mathbf{f}(y, u(y)), & \text{for } y \in \Omega, \\ u(y) = 0, & \text{for } ||y|| = 1, \\ \lim\limits_{||y|| \to \infty} u(y) = 0. \end{cases} \tag{4}$$

Investigation of the existence of radial solutions for (4) leads to the Dirichlet problem for the ODE of the form:

$$\begin{cases} -x''(t) = g(t, x(t)), & \text{on } (0,1), \\ x(0) = x(1) = 0, \end{cases} \tag{5}$$

with
$$g(t, v) = \frac{1}{(n-2)^2}(1-t)^{\frac{2n-2}{2-n}}\mathbf{f}((1-t)^{\frac{1}{2-n}}, v).$$

It is due to the fact that if $u(y) = z(||y||)$ with $z : [1, \infty) \to R$, is a radial solution of (4) then $x(t) = z\left((1-t)^{\frac{1}{2-n}}\right)$ satisfies (5) and for a solution x of (5), $u(y) = x(1 - ||y||^{2-n})$ satisfies (4). Now we recall the two relevant theorems from [10]:

Theorem 2. *Assume that there exist* $u_1, u_2 \in (0, +\infty)$ *such that* $\mathbf{f} : [1, +\infty) \times I \to [0, +\infty)$ *is continuous with* $I = (-u_1, u_2)$, $\mathbf{f}(t, \cdot)$ *is nondecreasing for* $t \in [1, +\infty)$, $\int_1^{+\infty} l^{n-1} \mathbf{f}(l, 0) dl \neq 0$ *and there exists* $0 < d < u_2$ *such that* $\frac{1}{n-2} \int_1^{\infty} l^{n-1} \mathbf{f}(l, d) dl \leq 4d$. *Then there exists at least one nontrivial solution* $\overline{x} \in X$ *of (5), with*

$$X = \left\{ x \in C^2((0,1)) \cap A_0 \ : \ x(t) \geq 0 \text{ for } t \in [0,1] \text{ and } x''(t) \leq 0 \text{ on } (0,1) \right. \\ \left. \text{and } ||x||_\infty \leq d \right\},$$

where A *denotes the space of absolutely continuous functions* $x : [0, 1] \to R$ *with* $x' \in L^2(0, 1)$ *and*

$$A_0 := \{ w \in A : \text{ such that } x(0) = x(1) = 0 \},$$

with the norm $\|x\|_{A_0} = \left(\int_0^1 |x'(t)|^2 \, dt \right)^{1/2}$.

Theorem 3. *Under the assumptions of the previous theorem the problem (4) possesses at least one nontrivial radial solution* $\overline{u} \in C(\overline{\Omega}) \cap C^2(\Omega)$ *such that* $0 \leq \overline{u}(y) \leq d$ *for* $y \in \Omega$.

Applying the above existence result for radial solution we obtain the following theorem:

Theorem 4. *Under the assumptions* (bp), (f1)–(f2) *there exists at least one nontrivial solution* u *of (1) such that* $0 \leq u(y) \leq d$. *If, in addition, the right-hand side of (1) is nonradial, then* u *has the same property.*

Proof. We start with the observation that we can extend $f \in C^1([0, d])$ to a function $\widetilde{f}$ belonging to $C^1(R)$. Let us consider (3) with $F : \Omega \times R \times R^n \to [0, +\infty)$ defined as follows

$$F(y, u, z) = p(y) \widetilde{f}(u) + b(y). \tag{6}$$

Taking into account conditions (bp) and (f1)–(f2) we can derive that conditions (D1) and (D2) from the Theorem 1 hold. To show that the last assumption of the Theorem 1 is satisfied, we use Theorem 3 which leads to the conclusion that there exists at least one nonnegative radial solution $\overline{u}$ (not identically equal to zero) for the problem

$$\begin{cases} -\Delta u = \overline{p}(||y||) f(u) + \overline{b}(||y||), & \text{for } y \in \Omega, \\ u(y) = 0, & \text{for } ||y|| = 1, \\ \lim_{||y|| \to \infty} u(y) = 0. \end{cases} \tag{7}$$

So that, by the definitions of $\overline{b}$ and $\overline{p}$, we can derive that

$$-\Delta \overline{u} \geq p(y) f(\overline{u}) + b(y),$$

which means that $\overline{u}$ is a supersolution of (3) with F given by (6). On the other hand $\underline{u}\equiv 0$ is a trivial subsolution of (3). Hence we infer that condition (D3) is also satisfied. Now, by Theorem 1, we obtain the existence of not necessarily radial solution u for (3) and $0 \leq u \leq \overline{u}$ in Ω. The last chain of inequalities and the fact that $\lim_{||y||\to\infty} \overline{u}(y) = 0$, imply assertion $\lim_{||y||\to\infty} u(y) = 0$. (It is clear that zero is not a solution for (1)). Finally, on account of above consideration we can derive that u is the nontrivial solution to the problem (1) and $0 \leq u \leq d$ for all $y \in \Omega$. □

Now we shall apply the above theorem to obtain the existence result for particular examples of (1). We show that the theory presented above is applicable to both sub- and superlinear problems with the right-hand side being smooth or nonsmooth function on $[0, +\infty)$.

Example 1. *Let us consider the following problem:*

$$-\Delta u = \left[\frac{1}{2\left\|y\right\|^5}\sum_{i=1}^{n}\cos(iy_i)\right]\left[\mathrm{e}^{\frac{-u^2}{u-8}} + u^2\right], \tag{8}$$

where $n = 3$, $\Omega := \left\{y = (y_1, y_2, y_3) \in R^3; \|y\| > 1\right\}$. *We observe that* $\overline{p}$ *and* f *given by*

$$\overline{p}(l) = \sup_{\|y\|=l}\left[\frac{1}{2\left\|y\right\|^5}\sum_{i=1}^{3}\cos(iy_i)\right] \leq \frac{3}{2l^5}$$

and

$$f(u) = \mathrm{e}^{\frac{-u^2}{u-8}} + u^2$$

satisfy the assumption (bp) and (f1) (for $I = (-1, 5)$*). Since*

$$f(3)\int_1^{+\infty} l^{n-1}\overline{p}(l)dl < 12,$$

we see that condition (f2) *holds for* $d = 3$. *So that we derive the existence of a nonradial solution* $\overline{u}$ *for (8) with the boundary condition as in (1). Of course,* $\overline{u}$ *is nonnegative function which is not identically equal to zero.*

Example 2. *The problem (1) with* Ω *defined as in Example 1 and* p *and* f *given by*

$$p(y) = \frac{y_1^4 + y_2^4 + y_3^2}{\|y\|^6},$$

$$f(u) = \frac{-u^4}{(u-9)(u+2)} + \ln(u + \mathrm{e})$$

possesses at least one nonradial solution. Indeed, it is clear that p *and* f *are smooth enough, namely they satisfy conditions* (bp) *and* (f1). *We also have*

$$\left[\int_1^{+\infty} l^{n-1}\overline{p}(l)dl\right] f(4) < 16,$$

which means that (f2) *holds (for* $d = 4$*). Finally, by Theorem 4, we obtain our claim.*

Let us remark that one cannot apply the theory presented in [12] *to both problems, since* f *is not smooth on* $[0, +\infty)$. *Moreover in the first example* f *is superlinear at infinity. Of course, our theory can be also used for* $f \in C^1(R)$.

Example 3. *We shall investigate (1) for* $f(u) = \frac{1}{5}[u^5 - u^4 + u^3 + u^2 - u + 1]$ *and* p *satisfying* (bp)*and the estimate* $\int_1^{+\infty} l^{n-1}\overline{p}(l)dl < 9$. *It is easy to see that assumptions* (f1) *and* (f2) *hold for* $d = 1$. *Thus we infer the existence of (at least one) positive solution to the BVP for the equation*

$$-\Delta u = \frac{p(y)}{5}\left[u^5 - u^4 + u^3 + u^2 - u + 1\right]$$

for $y \in \Omega$.

3. Continuous dependence on parameters

Now we are dealing with the stability of the following sequence

$$\text{(9)} \qquad \begin{cases} -\Delta u(y) = p(y)f(u(y)) + b_m(y), & \text{for } y \in \Omega, \\ u(y) = 0, & \text{for } ||y|| = 1, \\ \lim_{||y|| \to \infty} u(y) = 0, \end{cases}$$

where $\{b_m\}_{m \in N}$ is a sequence of functions from the space $C^{0,r}(\Omega)$, p and f satisfy conditions (**bp**) and (**f1**) and

(f2') for each $m \in N$ there exists $d_m \in R$ such that $0 < d_m < u_2$ and

$$\frac{1}{n-2}\left[f(d_m)\int_1^{+\infty} l^{n-1}\overline{p}(l)dl + \int_1^{+\infty} l^{n-1}\overline{b}_m(l)dl\right] \le 4d_m,$$

where $\overline{p}(l) = \sup_{||y||=l} p(y)$ and $\overline{b}_m(l) = \sup_{||y||=l} b_m(y)$ for all $l \ge 1$.

We investigate the behaviour of sequences of solutions $\{u_m\}_{m \in N}$ for (9) corresponding to the sequence of parameters $\{b_m\}_{m \in N}$ in the case, when $\{b_m\}_{m \in N}$ tends pointwise to a certain element $b_0 \in C^{0,r}(\Omega)$. We are also looking for the information about the links between the sequence $\{u_m\}_{m \in N}$ and a solution of the limit problem containing b_0. Our first task is to prove the continuous dependence on functional parameters of the solutions for the Dirichlet problem (5) corresponding to (7).

Theorem 5. *Assume hypotheses* (bp), (f1), (f2') *and suppose that* $\{b_m\}_{m\in N}$ *converges pointwise to* 0. *Then for each* $m \in N$ *there exists a solution* $x_m \in X_m$ *of (5) depending on* b_m, *namely*

$$-x''_m(t) = g_m(t, x_m(t)), \tag{10}$$

for $t \in (0,1)$, *where*

$$g_m(t,v) = \frac{1}{(n-2)^2}(1-t)^{\frac{2n-2}{2-n}}\left[\overline{p}((1-t)^{\frac{1}{2-n}})f(v) + \overline{b}_m((1-t)^{\frac{1}{2-n}})\right]$$

and

$$X_m = \{x \in C^2((0,1)) \cap A_0 \;:\; x(t) \geq 0 \text{ for } t \in [0,1] \text{ and } x''(t) \leq 0 \text{ on } (0,1) \text{ and } ||x||_\infty \leq d_m\}.$$

Moreover there exists $x_0 \in X_0$ *a solution of the following equation*

$$-x''(t) = g_0(t, x(t)) \quad \text{for } t \in (0,1),$$

such that (up to a subsequence) $\{x_m\}_{m\in N}$ *tends uniformly to* $x_0 \in C([0,1]) \cap C^2((0,1))$.

Proof. For every $m \in N$, Theorem 2 leads to the existence of $x_m \in X_m$ satisfying (10). Now we are going to investigate the properties of the sequence $\{x_m\}_{m\in N}$. By (f2') we derive that for each $m \in N$ and $t \in [0,1]$: $0 \leq x_m(t) \leq d_m \leq u_2$. Taking into account (10) and the fact that (3) implies the estimate

$$\int_0^1 g_m(t, d_m)dt \leq 4d_m,$$

we have

$$\int_0^1 [x'_m(t)]^2 dt = \int_0^1 x_m(t) g_m(t, x_m(t))dt \leq \int_0^1 d_m g_m(t, d_m)dt \leq 4(u_2)^2. \tag{11}$$

Summarizing, there exists $x_0 \in C([0,1])$ such that (up to a subsequence) $x_m \underset{m\to\infty}{\rightrightarrows} x_0$ uniformly. Let us consider the sequence $\{q_m\}_{m\in N}$ given by

$$q_m(t) = x'_m(t) \text{ on } (0,1).$$

Using (10) we obtain the pointwise convergence of $\{q'_m\}_{m\in N}$

$$\lim_{m\to\infty} q'_m(t) = \lim_{m\to\infty} -g_m(t, x_m(t)) = -g_0(t, x_0(t)) \tag{12}$$

on $(0,1)$ and the boundedness of $\{q'_m\}_{m\in N}$ in $L^1(0,1)$ norm and, by the definition of q_m and (11), the boundedness of $\{q_m\}_{m\in N}$ in $L^2(0,1)$. Hence

$\{q_m\}_{m\in N}$ (up to a subsequence) tends uniformly to a certain $\overline{q} \in C((0,1))$ such that $\overline{q}'(t) = -g(t, x_0(t))$ on $(0,1)$. Finally $\overline{q} \in C^1((0,1))$.

Taking into account the above reasoning we infer the equality

$$\overline{q}(t) = x_0'(t)$$

and further $x_0' \in C^1((0,1))$ and $x_0''(\cdot) = \overline{q}'(\cdot) = -g(\cdot, x_0(\cdot)) \leq 0$ on $(0,1)$, what we have claimed. □

As an immediate consequence of the above theorem we infer what follows

Theorem 6. *Suppose that conditions* (bp), (f1)–(f2') *hold and* $\{b_m\}_{m\in N}$ *tends pointwise to* 0. *Then for each* $m \in N$ *problem (7) (with nonlinearity containing* $\overline{b}_m(l) = \sup_{||y||=l} b_m(y)$*) possesses at least one radial solution* $\overline{u}_m$

$$\begin{cases} -\Delta \overline{u}_m = \overline{p}(||y||)\left[f(\overline{u}_m) + \overline{b}_m(||y||)\right], & \text{for } y \in \Omega \\ u(y) = 0, & \text{for } ||y|| = 1, \\ \lim_{||y||\to\infty} u(y) = 0. \end{cases}$$

Moreover there exists $\overline{u}_0$ *a solution of the following equation*

$$-\Delta u = \overline{p}(||y||) f(u) \quad \text{for } y \in \Omega$$

(with the same boundary conditions) such that (up to a subsequence) $\{\overline{u}_m\}_{m\in N}$ *tends uniformly to* $\overline{u}_0 \in C(\overline{\Omega}) \cap C^2(\Omega)$, *(*$\overline{p}(l) = \sup_{||y||=l} p(y)$*).*

Example 4. *Using the above theorem we can investigate the behaviour of a sequence of solutions for the following system of equations*

$$(13) \qquad \begin{cases} -\Delta u = \overline{p}(||y||)\left(\mathrm{e}^{\frac{-[u]^2}{u-6}} + u^3\right) + b_m(||y||), \\ u(y) = 0, \quad \text{for } ||y|| = 1, \\ \lim_{||y||\to\infty} u(y) = 0, \end{cases}$$

for $y \in \Omega$ *and* $\overline{p}(t) = \frac{1}{5}t^{-4}$, *for* $t \in [1, +\infty)$, $b_m(t) = \frac{\cos(t^2)}{mt^4}$ *for* $t \geq 1$ *and all* $m \in \mathbf{N}$. *Indeed, we can state that for* $f(u) = \mathrm{e}^{\frac{-[u]^2}{u-6}} + u^3$ *conditions* (f1)–(f2') *(with* $d = 3$*) and* (bp) *hold. Theorem 6 leads to the conclusion that for each* $m \in \mathbf{N}$ *the exists at least one nontrivial solution* $\overline{u}_m \geq 0$ *of (13). Finally, taking into account the uniform convergence of* $\{b_m\}_{m\in N}$ *to zero, we infer that the problem*

$$\begin{cases} -\Delta u = \overline{p}(||y||)\left(\mathrm{e}^{\frac{-u^2}{u-6}} + u^3\right), \\ u(y) = 0, \quad \text{for } ||y|| = 1, \ \lim_{||y||\to\infty} u(y) = 0, \end{cases}$$

possesses at least one nontrivial radial solution $\overline{u}_0$ *such that* $\{\overline{u}_m\}_{m\in N}$ *tends uniformly to* $\overline{u}_0$.

Applying Theorems 6 and 4 we obtain some results concerning the dependence on parameters also in nonradial case.

Corollary 1. *Assume that* p *is nonradial and that conditions* (bp), (f1)–(f2') *hold. If* $\{b_m\}_{m\in N} \subset C^{0,r}(\Omega)$ *is a sequence converging pointwise to* 0, *then for each* $m \in N$ *there exists a nontrivial solution* u_m *(not necessary radial) for the following PDE*

$$(14)\qquad \begin{cases} -\Delta u(y) = p(y)f(u(y)) + b_m(y), \\ \qquad u(y) = 0, \quad \text{for } ||y|| = 1, \lim\limits_{||y||\to\infty} u(y) = 0, \end{cases}$$

such that $0 \le u_m(y) \le \overline{u}_m(y)$ *for* $y \in \Omega$. *Moreover there exists a solution* u_0 *of the following equation*

$$(15)\qquad -\Delta u(y) = p(y)f(u(y)) \quad \text{for } \in \Omega$$

with the same boundary conditions and

$$(16)\qquad 0 \le \limsup_{m\to+\infty} u_m(y) \le \overline{u}_0(y) \quad \text{and} \quad 0 \le u_0(y) \le \overline{u}_0(y),$$

for $y \in \Omega$, *where (for each* $m \in N$*)* $\overline{u}_m$ *and* $\overline{u}_0$ *are supersolutions for (14) and (15), respectively, and* $\overline{u}_m \underset{m\to\infty}{\rightrightarrows} \overline{u}_0$.

Proof. Using Theorem 3.2 we can derive that for each $m \in N$ there exist: a supersolution $\overline{u}_m$ for (14) and a supersolution $\overline{u}_0$ for (15) such that $\overline{u}_m \underset{m\to\infty}{\rightrightarrows} \overline{u}_0$. Now Theorem 4 gives (for each $m \in N$) the existence of u_m and u_0 solutions for (14) and (15), respectively. Moreover

$$u_m(y) \le \overline{u}_m(y) \text{ and } u_0(y) \le \overline{u}_0(y)$$

in Ω. Both assertions yield (16). □

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